From Large to Small $\mathcal{N} = (4, 4)$ Superconformal Surface Defects in Holographic 6d SCFTs

Based on 2402.11745

Benjamin Suzzoni
In collaboration with P. Capuozzo, J. Estes and B. Robinson

Under the supervision of A. O'Bannon

May 2024

Table of Contents

Motivations

Why 2d defects in 6d SCFTs? (non-exhaustive list)

1. The 6d $\mathcal{N} = (2,0)$ is *special* (eg. Class-$\mathcal{S}$ and AGT, no known Lagrangian, strongly coupled...)
2. It admits 2d and 4d supersymmetric extended operators (defects)
3. Defects are needed to fully characterise the CFT (eg. order parameters in 4d, differentiation between gauge theories with same algebra but different groups, ...)
4. The tools are readily available (laziness argument)

Why holographic entanglement entropy?

1. Entanglement entropy of a region around the defect $\to$ universal information about the defect theory

Motivations

Holographically we can use:

1. Supergravity solutions with AdS_7 asymptotic regions
2. Systems of M5-branes in M-theory
3. Systems of M5-M2 brane intersections
4. All of the above (Asymptotically locally AdS_7 solutions to 11d supergravity)

Holographically, we can:

1. Use the Ryu-Takayanagi prescription to compute the entanglement entropy of a region in the 6d CFT
2. Use holographic renormalization to determine $\langle T \rangle$

Zooming into the landscape of 11d supergravity solutions describing codimension-4 defects in 6d SCFTs:

[D'Hoker et al.]

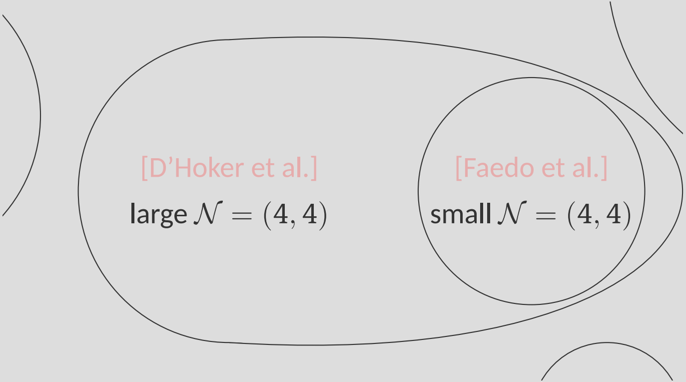
large $\mathcal{N} = (4,4)$

[Faedo et al.]

small $\mathcal{N} = (4,4)$

Objectives

Zooming into the landscape of 11d supergravity solutions describing codimension-4 defects in 6d SCFTs:



Objectives

1. Make precise the identification in the previous slide
2. Compute a holographic entanglement entropy
3. Extract Weyl anomaly coefficients
4. Compute stress-energy tensor one-point function $\langle T \rangle$
5. Identify A-type and B-type Weyl anomaly coefficients separately
6. Celebrate

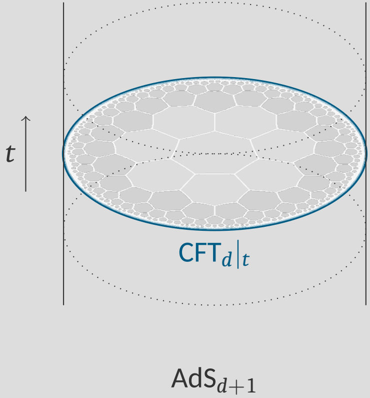
Objectives

1. Make precise the identification in the previous slide
2. Compute a holographic entanglement entropy
3. Extract Weyl anomaly coefficients
4. ~~Compute stress-energy tensor one-point function~~ $\cancel{\langle T \rangle}$
5. ~~Identify a-type and b-type Weyl anomaly coefficients separately~~
6. Still celebrate

- Motivations, objectives and outline
- Review of large $\mathcal{N} = (4, 4)$ surface defect solutions
- New small $\mathcal{N} = (4, 4)$ surface defect solutions
- Holographic entanglement entropy
- Holographic renormalization applied to $\langle T \rangle$ (and why it fails)
- Summary, open questions, and future directions

Table of Contents

The holographic principle (hand wavy)



- The bulk is a (quantum) gravity theory
- The CFT “lives” on the boundary
- The theories are **DUAL**

  (all n-point functions match)
- In the low energy limit: Supergravity $\leftrightarrow$ CFT

In a series of papers, [0806.0605], [0810.4647], [0906.0596], D'Hoker, Estes, Gutperle and Krym constructed a family of half-BPS solutions to 11d supergravity.

In other words, solutions to

$$R_{MN} - \frac{1}{12} F_{MPQR} F_N{}^{PQR} + \frac{1}{144} G_{NM} F_{PQRS} F^{PQRS} = 0 \tag{1}$$

$$d \star F + \frac{1}{2} F \wedge F = 0 \tag{2}$$

which preserve 16 real supercharges.

We will be looking at the slightly more recent formulation found in the followup paper Bachas et al. [1312.5477].

We are looking at solutions of 11d supergravity, with isometry superalgebra $\mathfrak{d}(2,1;\gamma) \oplus \mathfrak{d}(2,1;\gamma)$, $\gamma \in \mathbb{R}$.

We identify the maximal bosonic subgroup of its real form

$$\mathfrak{d}(2,1;\gamma;0) \supset \mathfrak{so}(2,1) \oplus \mathfrak{so}(3) \oplus \mathfrak{so}(3) \tag{3}$$

The geometry must thus contain

$$\mathrm{AdS}_3 \times S^3 \times \tilde{S}^3$$

fibered over a Riemann surface, Σ_2.

The 11d generalized Killing spinor equation

$$\nabla\epsilon + \frac{1}{288}\left(\Gamma_m{}^{npqr}e^m - 8\Gamma^{pqr}e^n\right)F_{npqr}\epsilon = 0 \tag{4}$$

is recast into a complex equation on Σ_2, for two functions $h, \mathcal{G} : \Sigma_2 \to \mathbb{C}$.

$$\partial\bar{\partial}h = 0 \qquad\qquad \partial\mathcal{G} = \frac{1}{2}(\mathcal{G} + \bar{\mathcal{G}})\partial\ln(h) \tag{5}$$

If Σ_2 has a boundary, we further impose the boundary conditions

$$h = 0 \qquad\qquad \mathcal{G} = \pm i \qquad\qquad \text{on } \partial\Sigma_2 \tag{6}$$

Explicitly, the non-zero supergravity fields are given by

$$G = f^2_{\text{AdS}_3} g_{\text{AdS}_3} + f^2_{S^3} g_{S^2} + f^2_{\tilde{S}^3} g_{\tilde{S}^3} + f^2_{\Sigma_2} g_{\Sigma_2} \tag{7}$$

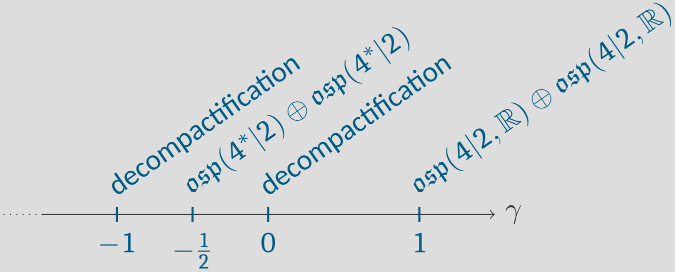
$$C_3 = b_{\text{AdS}_3} vol_{\text{AdS}_3} + b_{S^3} vol_{S^3} + b_{\tilde{S}^3} vol_{\tilde{S}^3} \tag{8}$$

where all coefficients depend only on $h, \mathcal{G}$ and γ (and $F = dC_3$).

So far, we take away:

- Fully specified by a triplet $(h, \mathcal{G}, \gamma)$
- $h, \mathcal{G} : \Sigma_2 \to \mathbb{C}$, where $\mathcal{G}$ is allowed to have singularities on $\partial\Sigma$
- Not all values of γ allow for asymptotic AdS solutions

Landscape of $\mathfrak{d}(2,1;\gamma) \oplus \mathfrak{d}(2,1;\gamma)$:



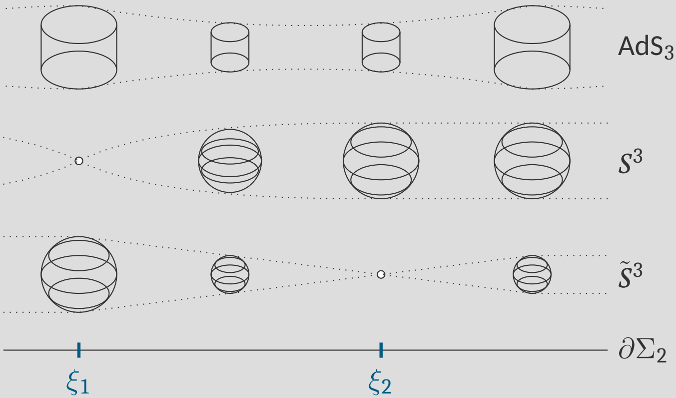
From here on, we choose Σ_2 to be the half-plane.

The most general solution with one AdS asymptotic region and $\gamma < 0$ is given by

$$h = -i(w - \bar{w}) \qquad \mathcal{G} = -i\left(1 + \sum_{j=1}^{2n+2} (-1)^j \frac{w - \xi_j}{|w - \xi_j|}\right) \tag{9}$$

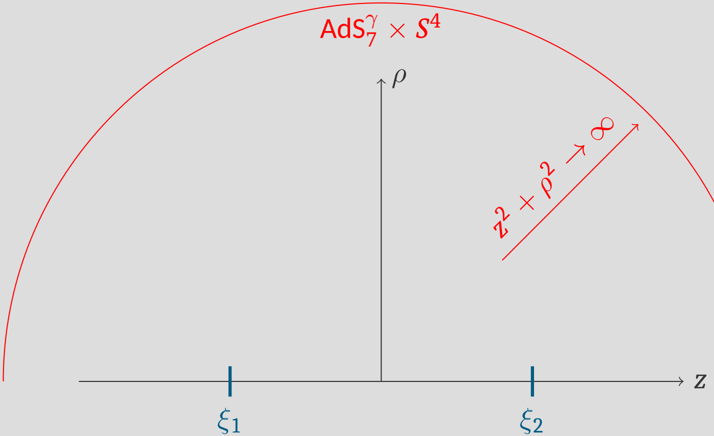
The simplest such solution, $n = 0$ gives $AdS_7 \times S^4$ (γ-deformed) globally.

All solutions exhibit the following behaviour on $\partial\Sigma_2$



The supergravity solution in Bachas *et al.* [1312.5477]

The $AdS_7 \times S^4$ (γ-deformed) region is found at large radius on the half-plane



Geometrical Derivation

Given the previous choice of $h, \mathcal{G}$, close to the boundary of AdS$_7$ (i.e. $v \to 0$)

$$G = \frac{4L^2}{v^2}\left(dv^2 + g_{AdS_3} + \frac{(1+\gamma)^2}{\gamma^2} g_{S^3}\right) + L^2\left(d\phi^2 + \sin^2(\phi) g_{\tilde{S}^3}\right) + \dots \tag{10}$$

$\hookrightarrow$ Asymptotically Locally "γ-deformed" AdS$_7 \times S^4$ metric.

Two ways of removing the deformation:

- $\gamma = -\frac{1}{2}$: Wilson surface defects in 6d SCFT
- $\gamma \to -\infty$: New surface defects

Geometrical Derivation

Let's not be too naïve

$$G = f^2_{\mathrm{AdS}_3}(h,\mathcal{G})g_{\mathrm{AdS}_3} + f^2_{S^3}(h,\mathcal{G})g_{S^2} + f^2_{\tilde{S}^3}(h,\mathcal{G})g_{\tilde{S}^3} + f^2_{\Sigma_2}(h,\mathcal{G})g_{\Sigma_2} \tag{11}$$

$\gamma \to -\infty$ is ill defined (interior blows up).

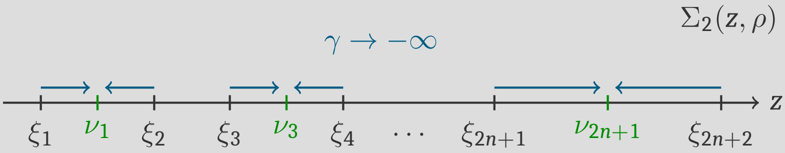
Solution: rescale the critical points of $\mathcal{G}$ accordingly,

$$\xi_j = \nu_j - \gamma^{-1}\hat{\xi}_j \qquad\qquad \nu_1 = \nu_2, \quad \nu_3 = \nu_4, \quad \text{etc} \tag{12}$$

They are now grouped in pairs, in the limit $\gamma \to -\infty$.

Geometrical Derivation

Taking the $\gamma \to -\infty$ limit



The general metric solution takes the form

$$G = h_1 H^{-\frac{1}{3}} \left(g_{\mathrm{AdS}_3} + g_{S^3}\right) + H^{\frac{2}{3}} \left(dz^2 + d\rho^2 + \rho^2 g_{\tilde{S}^3}\right) \,, \tag{13}$$

where

$$H(z,\rho) = \frac{h_1}{2} \sum_{j=1}^{2n+2} \frac{(-1)^j \hat{\xi}_j}{(\rho^2 + (z-\nu_j)^2)^{3/2}} \,. \tag{14}$$

$$G = h_1 H^{-\frac{1}{3}} \left(g_{\mathrm{AdS}_3} + g_{S^3}\right) + H^{\frac{2}{3}} \left(dz^2 + d\rho^2 + \rho^2 g_{\tilde{S}^3}\right) \tag{15}$$

is precisely the metric part of the solution proposed by Faedo *et al.* [2007.16167]

Advantage of our formulation:

- Found a large set of H that solve the e.o.m.
- Geometry is Ricci-finite
- Allows for an identification of the 2d defect rep. from its Young tableau

All in all, the supergravity solution is fully specified by the following data,

$$G = h_1 H^{-\frac{1}{3}} \left(g_{\mathrm{AdS}_3} + g_{S^3}\right) + H^{\frac{2}{3}} \left(dz^2 + d\rho^2 + \rho^2 g_{\tilde{S}^3}\right) \tag{16}$$

$$F = 2h_1 vol_{\mathrm{AdS}_3} \wedge dz + 2h_1 vol_{S^3} \wedge dz + \partial_z H \rho^3 d\rho \wedge vol_{\tilde{S}^3} - \partial_\rho H \rho^3 dz \wedge vol_{\tilde{S}^3} \tag{17}$$

$$H = \frac{h_1}{2} \sum_{j=1}^{2n+2} \frac{(-1)^j \hat{\xi}_j}{(\rho^2 + (z - \nu_j)^2)^{3/2}} \tag{18}$$

The new small $\mathcal{N} = (4,4)$ solution

What does this solution actually describe?

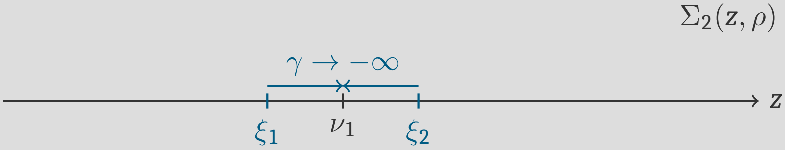
$$\gamma = -\tfrac{1}{2}: \qquad \mathfrak{d}(2,1;-\tfrac{1}{2};0) \oplus \mathfrak{d}(2,1;-\tfrac{1}{2};0) = \mathfrak{osp}(4^\star|2) \oplus \mathfrak{osp}(4^\star|2) \subset \mathfrak{osp}(8^\star|4)$$

Generically it the algebra is not a subalgebra of the 6d $\mathcal{N} = (2,0)$!

The theory must be deformed somehow... We must be careful when choosing what to call the *vacuum*!

The 1PT vacuum

The simplest configuration we can consider is $n = 0$, a single singularity in H



which is the $\gamma \to -\infty$ limit of the γ-deformed $\mathrm{AdS}_7 \times S^4$ vacuum.

We call this the *1PV vacuum*.

Table of Contents

Weyl anomaly coefficients

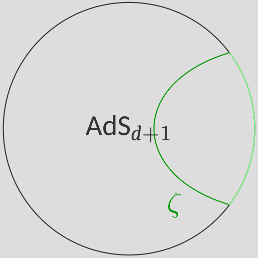
Conformal symmetry breaking is characterised by the Weyl anomaly coefficients,

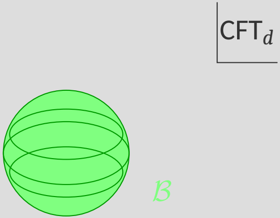
$$\langle T^{\mu}{}_{\mu} \rangle \simeq a R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - d_2 W_{\mu\nu} W^{\mu\nu} + \cdots \tag{19}$$

Including defects we get also get a contribution localized on the defect as

$$T = T_{\text{bulk}} + \delta T_{\text{defect}} \tag{20}$$

For a CFT with a weakly-coupled static gravity dual the RT formula allows us to recast the field theory computation of S_{EE} into a Plateau problem in the bulk,





$$S_{EE} = \min_{\zeta} \frac{\mathcal{A}[\zeta]}{4G_N}$$

$$S_{EE} = -\mathrm{tr}_{\mathcal{B}}(\rho_{\mathcal{B}} \ln(\rho_{\mathcal{B}}))$$

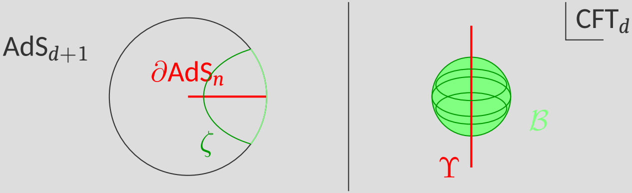
When $\mathcal{B} = B^{d-1}(l)$ is a ball of radius l in CFT$_d$,

$$S_{EE} = \begin{cases} p_1 \left(\frac{l}{\epsilon}\right)^{d-2} + \cdots + p_{d-2}\frac{l}{\epsilon} + p_{d-1} + o(1) & d\text{: odd} \\ p_1 \left(\frac{l}{\epsilon}\right)^{d-2} + \cdots + p_{d-3}\left(\frac{l}{\epsilon}\right)^2 + q\ln\left(\frac{l}{\epsilon}\right) + o(1) & d\text{: even} \end{cases} \tag{21}$$

eg. $d = 2$, $S_{EE} = \frac{c}{3}\ln\left(\frac{l}{a}\right)$

In general, q or p_{d-1} contain information about the Weyl anomaly coefficients of the CFT.

Including defects



In general, properly extracting out defect contributions is *hard*.
The solution, subtracting out the contribution from the ambient CFT,

$$q_\Upsilon = l\frac{d}{dl}\left(S_{EE}[\Upsilon] - S_{EE}\right) \tag{22}$$

Importantly, for a 2d defect in a 6d ambient CFT

$$q_\Upsilon = \frac{1}{3}\left(a_\Upsilon + \frac{3}{5}d_2\right) \tag{23}$$

where

$$\langle T^\mu{}_\mu\rangle_\Upsilon \simeq a_\Upsilon R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - d_2 W_{\mu\nu}W^{\mu\nu} + \cdots \tag{24}$$

(Note that more terms may appear here than for a standalone 2d CFT one-point function)

Applying the RT prescription on the new supergravity solutions, with $S_{EE} =$1PV, we find

$$q_\Upsilon = -\frac{(\varpi,\varpi)}{5} = \frac{1}{3}\left(a_\Upsilon + \frac{3}{5}d_2\right) \tag{25}$$

where ϖ is the highest weight of the irrep. describing the defect.

Can we now separate out a_Υ and d_2?

Asymptotic solutions to Einstein's equation $R_{\mu\nu} - \frac{1}{2} R G_{\mu\nu} = \Lambda G_{\mu\nu}$ may be written in Fefferman-Graham (FG) form

$$G = l^2 \left(\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g \right) \tag{26}$$

$$g = g_{(0)} + \cdots + \rho^{d/2} g_{(d)} + f_{(d)} \rho^{d/2} \ln(\rho) + \cdots \tag{27}$$

and obey

$$\rho[2g'' - 2g' g^{-1} g' + \mathrm{Tr}(g^{-1} g') g'] + \mathrm{Ric}(g) - (d-2) g' - \mathrm{Tr}(g^{-1} g') g = 0 \tag{28a}$$

$$\nabla_i \mathrm{Tr}(g^{-1} g') - \nabla^j g'_{ij} = 0 \tag{28b}$$

$$\mathrm{Tr}(g^{-1} g'') - \frac{1}{2} \mathrm{Tr}(g^{-1} g' g^{-1} g') = 0 \tag{28c}$$

$$g = g_{(0)} + \rho g_{(2)} + \rho^2 g_{(4)} + \rho^3 g_{(6)} + f_{(6)} \rho^3 \ln(\rho) + \cdots$$

For Asymptotically locally AdS$_7$ solutions, we can solve for $g_{(2)}$, $g_{(4)}$ and $f_{(6)}$ in terms of $g_{(0)}$.

The final component, $g_{(6)}$ is related to the stress-energy tensor

$$\langle T_{ij} \rangle = \frac{3}{8\pi G_N} \left(g_{(6)ij} - A_{(6)ij} + \frac{1}{24} S_{ij} \right) \tag{29}$$

$\hookrightarrow$ source of Weyl anomaly coefficients

What about AlAdS$_7 \times S^4$ solutions?

What about AlAdS$_7 \times S^4$ solutions?

Start by writing the 11d metric as a perturbation around AdS$_7 \times S^4$,

$$G = g_{\mathrm{AdS}_7} \oplus g_{S^4} + h_{11} \tag{30}$$

and dimensionally reduce on the S^4

$$g_7 = \left(1 + \frac{\bar{\zeta}}{5}\right) g_{\mathrm{AdS}_7} + \bar{h}_7 \tag{31}$$

Only 0-modes of the internal space contribute.

Recall that our 11d metric is AlAdS$_7 \times S^4$ and can be written in the FG gauge

$$g = \frac{L^2}{\rho^2}\left(d\rho^2 + 4\rho\, g_6\right) \oplus L^2 g_4 \tag{32}$$

while the four-form flux

$$\begin{aligned} F =& -\frac{8L^3\cos(\phi)}{\rho^2} d\rho \wedge \left(vol_{\mathrm{AdS}_3} + vol_{S^3}\right) - \frac{8L^3\sin(\phi)}{\rho} d\phi \wedge \left(vol_{\mathrm{AdS}_3} + vol_{S^3}\right) \\ &+ 3L^3\sin^3(\phi) d\phi \wedge vol_{\tilde{S}^3} + o(1) \end{aligned} \tag{33}$$

Asymptotically close to the boundary, $\rho\to 0$, the supergravity e.o.m. may be recast into equations for g_6 and g_4. For example, the (ρ,ρ) component is

$$\frac{1}{2}\mathrm{Tr}(g_6^{-1}g_6'g_6^{-1}g_6') - \mathrm{Tr}(g_6^{-1}g_6'') + (g_6\leftrightarrow g_4) - \frac{d}{2\rho^2} - \frac{1}{\rho}\mathrm{Tr}(g_4^{-1}g_4')$$
$$= -\frac{1}{6}F_{\rho MNP}F_\rho{}^{MNP} - \frac{1}{72}\frac{L^2}{\rho^2}F_{MNPQ}F^{MNPQ} \tag{34}$$

This can be used to solve $g_{6(2)}$ in terms of $g_{6(0)}, g_{4(0)}, g_{4(2)}$ and F etc.

Problem: F doesn't have the right asymptotics to recover [de Haro]'s equations in 7d.

Table of Contents

Summary of Results

- Identified the supergravity solution from Faedo *et al.* [2007.16167] as the limit of Bachas *et al.* [1312.5477]
- Found new explicit solutions to the e.o.m. of the former (Ricci finite)
- Computed the holographic entanglement entropy of a region around the defect and extracted out a linear combination of Weyl anomaly coefficients
- (not shown here) Established the defect Young tableau - allowing for an identification of the M2/M5-brane charges

Open Questions

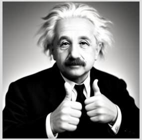
- Asymptotically, 4-form flux has YT-independent leading-order term
  - $\hookrightarrow$ supergravity solution doesn't admit global AdS$_7 \times S^4$ configurations
  - $\hookrightarrow$ no way of "removing" the defect
  - $\hookrightarrow$ isometry superalgebra isn't a subalgebra of $\mathfrak{osp}(8^*|4)$

$\Longrightarrow$ The $\mathcal{N} = (2, 0)$ theory is deformed... by what?

- Standard holographic renormalization techniques can't be applied here. How can we fix this?

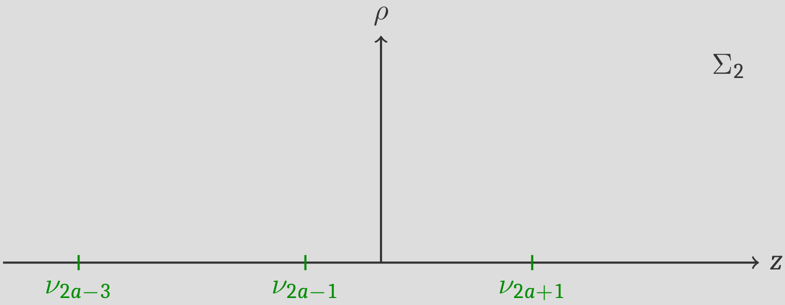
From Large to Small $\mathcal{N} = (4, 4)$ Superconformal Surface Defects in Holographic 6d SCFTs

ありがとうございました

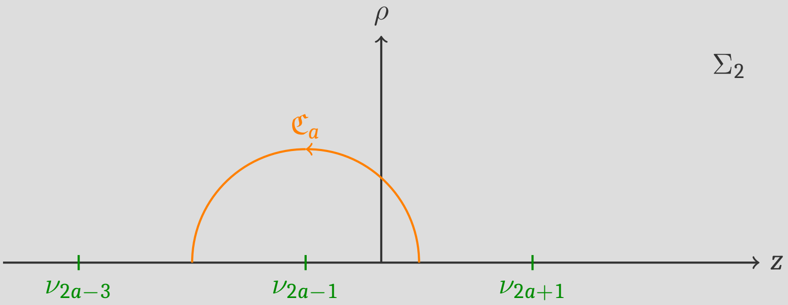


Young tableaux

6 Summary, open questions, and future directions

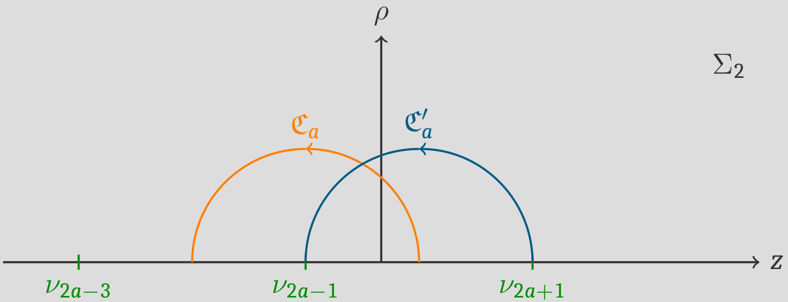


(35)



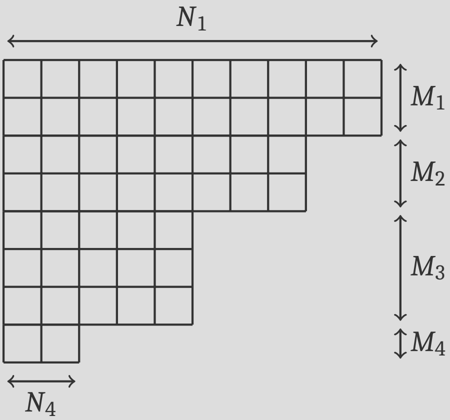
$$M_a = \frac{1}{2(4\pi^2 G_N)^{1/3}} \int_{\mathfrak{C}_a} P_{\mathfrak{C}_a}[\mathcal{F}_{(4)}] \tag{35}$$

Young tableaux



$$M_a = \frac{1}{2(4\pi^2 G_N)^{1/3}} \int_{\mathfrak{C}_a} P_{\mathfrak{C}_a}[\mathcal{F}_{(4)}] \qquad M'_a = \frac{1}{2(4\pi^2 G_N)^{1/3}} \int_{\mathfrak{C}'_a} P_{\mathfrak{C}'_a}[\mathcal{F}_{(4)}] \tag{35}$$

Young tableaux



Abstract

In 11d supergravity, there are known solutions with superisometry algebra d(2,1;γ)+d(2,1;γ) which are holographically dual to the 6d maximally superconformal field theory with 2d superconformal defects. In this talk I will present our work from 2402.11745 in which we show that a limit of these solutions, for which $\gamma \to -\infty$, reproduces another known class of solutions, holographically dual to small N=(4,4) superconformal surface defects. Notably, our construction produces a finite Ricci scalar, in contrast to the known small N=(4,4) solutions. Using the Ryu-Takayanagi prescription, one can calculate the Entanglement Entropy of a spherical region centered around the defect. I will show how this was done for these new small N=(4,4) solutions, allowing us to extract a linear combination of defect Weyl anomaly coefficients. Finally, I will argue why the standard stress-energy tensor one-point function computation can't be done here, explaining why we can't separate out the A-type and B-type coefficients using these computations alone.