An exact interpolating index for 4d $\mathcal{N} = 2$ SCFTs

[2411.xxxxx] (hopefully)

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Trace definition

The Superconformal Index (SCI) is defined as the Witten index of the theory in radial quantization [Gadde, Rastelli, Razamat, Yan 2013]

$$\mathcal{I}(\mu_i) = \mathrm{Tr}_{\mathcal{H}(S^{d-1})}\left((-1)^F e^{-\beta\delta} \prod_{i=1}^{N} \mu_i^{\mathcal{M}_i}\right) \tag{1}$$

where $\delta = 2\left\{\mathcal{Q}, \mathcal{Q}^\dagger\right\}$.

Counts states annihilated by δ and is independent of β.

The 4d $\mathcal{N} = 2$ superconformal algebra has 8 real (Q-)supercharges.

$$\mathcal{Q}_{1-} \quad \mathcal{Q}_{1+} \quad \mathcal{Q}_{2-} \quad \mathcal{Q}_{2+} \quad \tilde{\mathcal{Q}}_{1\dot{-}} \quad \tilde{\mathcal{Q}}_{1\dot{+}} \quad \tilde{\mathcal{Q}}_{2\dot{-}} \quad \tilde{\mathcal{Q}}_{2\dot{+}} \tag{2}$$

Choose, say, $\tilde{\mathcal{Q}}_{1\dot{-}}$, to compute the index.

The commutants of $\tilde{\delta}_{1\dot{-}}$ are: $\tilde{\delta}_{2\dot{+}}$, δ_{1+} and δ_{1-}.

$$\mathcal{I}(p,q,t) = \mathrm{Tr}_{\mathcal{H}(S^{d-1})}\left((-1)^F p^{\frac{1}{2}\delta_{1+}} q^{\frac{1}{2}\delta_{1-}} t^{R+r} e^{-\beta\tilde{\delta}_{1\dot{-}}}\right) \tag{3}$$

Superconformal Index

$\mathcal{I}(p,q,t)$

$\tau \to 0$

$p = 0$

$q = t$

Coulomb-branch Index

$\mathcal{I}(\tau\sigma, \tau\rho, \tau^2)$

$\hookrightarrow$ dep. on ρ, σ only

Macdonald Index

$\mathcal{I}(0,q,t)$

$q = 0$

$q = t$

Hall-Littlewood Index

$\mathcal{I}(0,0,t)$

Schur Index

$\mathcal{I}(p,q,q)$

Partition function definition

The SCI may also be defined as a partition function on $S^3 \times S^1_\beta$

$$Z_{S^3 \times S^1_\beta} = e^{-\beta E_{\text{Casimir}}} \mathcal{I} \tag{4}$$

The various deformations around the maximal supersymmetric configuration (i.e. round $S^3 \times S^1$) parameterize p, q and t.

$$\mathbb{R}^4 \quad \xrightarrow[\text{equiv.}]{\text{conf.}} \quad S^3 \times \mathbb{R} \quad \xrightarrow[\text{holo.}]{U(1)_R} \quad S^3 \times S^1_\beta$$

Can define many indices in that way $\hookrightarrow L(p,q) \times S^1$, half index, etc

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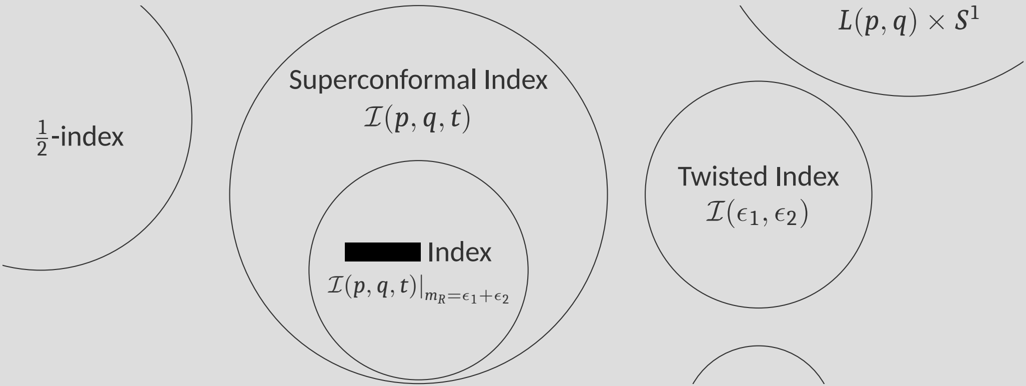
Objectives

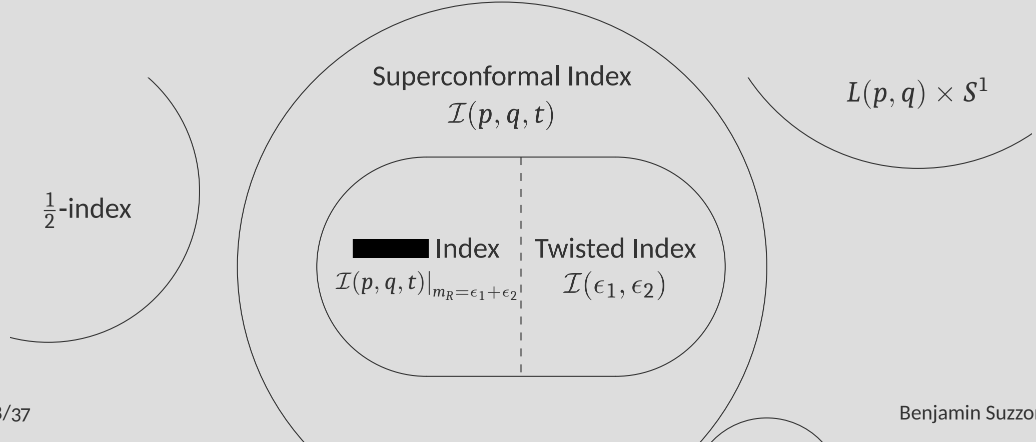
2 What is this talk about?

Zooming into the landscape of indices of 4d $\mathcal{N} = 2$ SCFTs



Objectives

Zooming into the landscape of indices of 4d $\mathcal{N}=2$ SCFTs



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Via minimal coupling

Consider a supersymmetric field theory on $(\mathbb{R}^d, \delta)$ with Lagrangian density $\mathcal{L}_{\mathbb{R}^d}$.

Can we place it on any Riemannian manifold $(\mathcal{M}_d, g)$?

Minimal coupling:

$\delta \mapsto g$	Couple stress-energy tensor to rank-2 background tensor field $h_{\mu\nu}$
$\partial \mapsto \nabla$	(Linear coupling + seagull terms)
$\mathcal{L} \mapsto \sqrt{g}\mathcal{L}$	$\mathcal{L}_{\mathcal{M}_d} = \mathcal{L}_{\mathbb{R}^d} - \frac{1}{2} h_{\mu\nu} T^{\mu\nu} + \mathcal{O}(h^2)$

Via minimal coupling

$$\mathcal{L}_{\mathcal{M}_d} = \mathcal{L}_{\mathbb{R}^d} - \frac{1}{2} h_{\mu\nu} T^{\mu\nu} + \mathcal{O}(h^2) \tag{5}$$

Does minimal coupling preserve supersymmetry on $\mathcal{M}_d$?

Ingredients:

1. $\mathcal{M}_d$ must be Spin (admit a Spin-bundle $\mathcal{S}$)
2. $\mathcal{M}_d$ must admit non-trivial solutions to $\nabla\xi = 0$, $\xi \in \Gamma(\mathcal{S})$

Answer: Yes but for very few such $\mathcal{M}_d$.
eg. 4d compact: only T^4 and K3-surfaces with Ricci-flat Kähler metrics

Via twisting

If our theory has large enough R-symmetry,

1. introduce a connection on R-symmetry bundle, A_R

Supersymmetry is now preserved iff

$$d\xi + \frac{1}{4}\omega_{ab}\gamma^{ab}\xi + \frac{1}{2}A_R \cdot \xi = 0 \tag{6}$$

2. identify R-symmetry bundle with sub-bundle of Spin-bundle (eg $A_R = \omega^+$)

Supersymmetry is now preserved iff

$$d\xi^+ = 0 \tag{7}$$

Via twisting

Twisting redefines the *Lorentz* group

$$\text{Lorentz}' \subset \text{Lorentz} \times \mathcal{R}$$

One of the original supercharges $\mathcal{Q}$ becomes a singlet of $\text{Lorentz}'$ and $T_{\mu\nu}$ becomes $\mathcal{Q}$-exact.

$$d_{\text{Met}} \int \mathcal{D}\Phi e^{-S_{\text{twisted}}[\Phi; g_{\mu\nu}]} = 0 \tag{8}$$

Content of theory decomposes into cohomology of $\mathcal{Q}$. We get a *cohomological TQFT*.

Ingredients: $\mathcal{M}_d$ can be any smooth manifold

The Donaldson-Witten twist

Case of interest here: 4d $\mathcal{N} = 2$

$$\begin{aligned} \mathrm{Spin}(4) \times SU(2)_R \times U(1)_R &\cong SU(2)_l \times SU(2)_r \times SU(2)_R \times U(1)_R \\ &\hookrightarrow SU(2)_l \times SU(2)' \times U(1)_R \end{aligned} \tag{9}$$

The partition function

$$Z_{\mathrm{DW}}[g_{\mu\nu}] = \int \mathcal{D}\Phi e^{-S_{\text{twisted}}[\Phi; g_{\mu\nu}]} \tag{10}$$

localizes to the volume of the moduli space of instantons.
$\hookrightarrow$ divergence can be regularized (eg. Ω-deformation/T^2-equivariant volume)

The systematic way

A la Festuccia-Seiberg [Festuccia, Seiberg 2011]

1. Couple theory to off-shell supergravity
2. Take the rigid limit ($M_P \to \infty$)

The Lagrangian density then looks like

$$\mathcal{L}_{\mathbb{M}_d} = \mathcal{L}_{\mathbb{R}^d} + (\mathcal{H}, \mathcal{S}) + \mathcal{O}(\Phi) \tag{11}$$

$\mathcal{H}$: sugra multiplet, $\mathcal{S}$: susy multiplet containing $T_{\mu\nu}$

Supersymmetry is conserved when $\delta_\xi \mathcal{H} = 0$.

The systematic way

Example: 4d $\mathcal{N} = 1$ [Closset, Dumitrescu, Festuccia, Komargodski 2014]

1. $\mathcal{R}$-multiplet $\rightarrow$ new minimal supergravity

$$\begin{aligned} \mathcal{R}_\mu &= (j_\mu^{(R)}, S_{\alpha\mu}, \tilde{S}^{\dot{\alpha}}{}_\mu, T_{\mu\nu}, \mathcal{F}_{\mu\nu}) \\ \mathcal{H}_\mu &= (A_\mu^{(R)}, \Psi_{\alpha\mu}, \tilde{\Psi}^{\dot{\alpha}}{}_\mu, h_{\mu\nu}, \mathcal{B}_{\mu\nu}) \\ \Delta\mathcal{L} &= 2\int d^4\theta \mathcal{R}_\mu \mathcal{H}^\mu \end{aligned} \tag{12}$$

2 Ferrara-Zumino multiplet $\rightarrow$ old minimal supergravity

When the theory is conformal, it is natural to couple to *conformal supergravity*.
With a 4d $\mathcal{N} = 2$ SCFT, we wish to couple to the Weyl multiplet of $\mathcal{N}$-extended conformal sugra [de Wit, Reys 2017]

$$e_\mu{}^a \quad \psi_\mu{}^i \quad b_\mu \quad A_\mu^{U(1)_R} \quad A_\mu^{SU(2)_R} \quad T_{ab} \quad \chi^i \quad D \tag{13}$$

$$\Delta\mathcal{L} = J_e^\mu{}_a\, e_\mu{}^a + \bar{J}^\mu_{\psi i}\, \psi_\mu{}^i + J^\mu_{U(1)_R}\, A_\mu^{U(1)_R} + \mathrm{tr}(J^\mu_{SU(2)_R}\, A_\mu^{SU(2)_R}) + J_T^{ab}\, T_{ab} + \bar{J}_{\chi i}\, \chi^i + J_D\, D \tag{14}$$

The currents $(J_e, J_\psi, J_{U(1)}, J_{SU(2)}, J_T, J_\chi, J_D)$ form the Sohnius multiplet. [Sohnius 1979]

Supersymmetry is preserved iff the variation of the fermionic fields of the Weyl multiplet vanish

$$\delta_{\xi,\eta}\psi_\mu{}^i = 2D_\mu\xi^i + \frac{\mathrm{i}}{16}T_{ab}\gamma^{ab}\gamma_\mu\xi^i - \mathrm{i}\gamma_\mu\eta^i = 0 \tag{15}$$

$$\begin{aligned}\delta_{\xi,\eta}\chi^i = {}& \frac{\mathrm{i}}{24}\gamma^{ab}\gamma^\mu D_\mu T_{ab}\xi^i + \frac{1}{6}R(A^{SU(2)_R})_{ab}{}^i{}_j\gamma^{ab}\xi^j - \frac{1}{3}R(A^{U(1)})_{ab}\gamma^{ab}\gamma^5\xi^i + D\xi^i \\ & + \frac{1}{24}T_{ab}\gamma^{ab}\eta^i = 0\end{aligned} \tag{16}$$

$\hookrightarrow$ generalizes Killing spinor equation

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Only “turn on” the following Weyl multiplet fields

$$\begin{aligned} ds^2 &= d\theta^2 + \sin^2(\theta)(d\varphi + \epsilon_1 dt)^2 + \cos^2(\theta)(d\tau + \epsilon_2 dt)^2 + \beta^2 dt^2 \\ A^{U(1)_R} &= (-\beta + \mathbb{i}(m_R - \epsilon_1 - \epsilon_2))dt \\ A^{SU(2)_R} &= -\mathbb{i} m_R \sigma_3 dt \end{aligned} \tag{17}$$

$$\begin{pmatrix} \mathrm{Spin}(4) \\ SU(2) \end{pmatrix} \supset \begin{pmatrix} U(1)^2 \\ U(1) \end{pmatrix} \ni \begin{pmatrix} \epsilon_1, \epsilon_2 \\ m_R \end{pmatrix} \longleftrightarrow \begin{pmatrix} p, q \\ t \end{pmatrix}$$

This background admits two generalized Killing spinors (+ η's)

$$(\xi_+)^{\alpha i} = (\sigma_1 - \mathbb{i}\sigma_2)^{\alpha i} \quad (\xi_-)^{\dot\alpha i} = (\sigma_1 + \mathbb{i}\sigma_2)^{\dot\alpha i} \tag{18}$$

The general SCI background preserves two Killing spinors. We can enhance this by specializing to

$$m_R = \epsilon_1 + \epsilon_2 \tag{19}$$

The background reduces to

$$\begin{aligned} ds^2 &= d\theta^2 + \sin^2(\theta)(d\varphi + \epsilon_1 dt)^2 + \cos^2(\theta)(d\tau + \epsilon_2 dt)^2 + \beta^2 dt^2 \\ A^{U(1)_R} &= -\beta dt \\ A^{SU(2)_R} &= -\mathbb{i}(\epsilon_1 + \epsilon_2)\sigma_3 dt \end{aligned} \tag{20}$$

which now preserves four Killing spinors

$$(\xi_+)^{\alpha i} = (\sigma_1)^{\alpha i} \quad (\xi_+)^{\alpha i} = (\sigma_2)^{\alpha i} \quad (\xi_-)^{\dot\alpha i} = (\sigma_1)^{\dot\alpha i} \quad (\xi_-)^{\dot\alpha i} = (\sigma_2)^{\dot\alpha i} \tag{21}$$

The twisted background

The topological twist may be implemented on the conformal supergravity by “turning on”

$$
\begin{aligned}
\mathcal{M}_4 &= S^3 \times S^1_\beta \\
A^{SU(2)_R} &= \frac{1}{2}\omega^{ab}\sigma_{ab} \\
D &= \frac{1}{6}R
\end{aligned}
\tag{22}
$$

This background preserves two constant Killing spinors

$$
(\xi_+)^{\alpha i} = (\sigma_2)^{\alpha i} \quad (\xi_-)^{\dot{\alpha} i} = (\sigma_2)^{\dot{\alpha} i} \tag{23}
$$

Now, $Z[g, A^{SU(2)_R}, D] = Z_{\mathrm{DW}}[S^3 \times S^1_\beta]$... but we can do more!

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After twisting, we are still free to turn on supergravity fields at no further cost in susy,

$$\begin{aligned} ds^2 &= d\theta^2 + \sin^2(\theta)(d\varphi + \epsilon_1 dt)^2 + \cos^2(\theta)(d\tau + \epsilon_2 dt)^2 + \beta^2 dt^2 \\ A^{SU(2)_R} &= \frac{1}{2}\omega^{ab}\sigma_{ab} \quad T = 4(dv)^+ \quad D = \frac{R}{6} \end{aligned} \tag{24}$$

where $v = \epsilon_1 \partial_\varphi + \epsilon_2 \partial_\tau$.

The background now preserves

$$\begin{pmatrix} (\xi_+)^{\alpha i} \\ (\xi_-)^{\dot\alpha i} \end{pmatrix} = \begin{pmatrix} (\sigma_2)^{\alpha i} \\ \mathrm{i} v^\mu (\sigma_\mu \sigma_2)^{\dot\alpha i} \end{pmatrix} \quad (\xi_-)^{\dot\alpha i} = (\sigma_2)^{\dot\alpha i} \tag{25}$$

$\hookrightarrow U(1)^2$-equivariant twisted index

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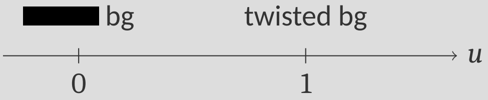
5 The Interpolating Index

The conformal supergravity background

5 The Interpolating Index

$$\begin{aligned}
&ds^2 = d\theta^2 + \sin^2(\theta)(d\varphi + \epsilon_1 dt)^2 + \cos^2(\theta)(d\tau + \epsilon_2 dt)^2 + \beta^2 dt^2 \\
&A^{U(1)_R} = \beta(u-1)dt \\
&A^{SU(2)_R} = \frac{u}{2}\omega^{ab}\sigma_{ab} + \mathbb{i}(u-1)(\epsilon_1+\epsilon_2)\sigma_3 dt \\
&T = 4u(dv)^+ \quad D = \frac{u(2-u)}{6}R
\end{aligned} \tag{26}$$

It interpolates between the [redacted] background and the twisted backgrounds



The conformal supergravity background

This background preserves one Killing spinor, common to the [redacted] and twisted backgrounds

$$(\xi_-)^{\dot{\alpha} i} = (\sigma_2)^{\dot{\alpha} i} \tag{27}$$

We will denote the corresponding partition function $\mathcal{I}(\epsilon_1, \epsilon_2; u)$

$$\mathcal{I}(\epsilon_1, \epsilon_2; u) = Z_{S^3 \times_\Omega S^1_\beta}[A^{U(1)_R}, A^{SU(2)_R}, T, D] \tag{28}$$

$$\begin{tikzcd} & {} \arrow[dl, "u=0"'] \arrow[dr, "u=1"] & \\ \mathcal{I}(p, q, t)|_{m_R = \epsilon_1 + \epsilon_2} & & \mathcal{I}_{\text{twist}}(\epsilon_1, \epsilon_2) \end{tikzcd}$$

We claim that the interpolation is *exact*

$$\frac{d\mathcal{I}}{du} = 0 \tag{29}$$

which in turn, means that we can equate the [redacted] index and the twisted index.

$$\mathcal{I}(p, q, t)|_{m_R=\epsilon_1+\epsilon_2} = \mathcal{I}_{\text{twist}}(\epsilon_1, \epsilon_2) \tag{30}$$

*Ignoring all factors of the supersymmetric Casimir energy

Proof: Step 0

5 The Interpolating Index

1. Pick bosonic fixed point $\overline{\mathcal{H}}$
2. Expand around fixed point $\mathcal{H} = \overline{\mathcal{H}} + t\Delta\mathcal{H}$

$$S[\Phi; \overline{\mathcal{H}} + t\Delta\mathcal{H}] = S[\Phi; \overline{\mathcal{H}}] + t \sum_A \int (J_A, \Delta\mathcal{H}_A) + \mathcal{O}(t^2) \tag{31}$$

where $\mathcal{H}_A \in ({e_\mu}^a, {\psi_\mu}^i, b_\mu, A_\mu^{U(1)_R}, A_\mu^{SU(2)_R}, T_{ab}, \chi^i, D)$ and $J_A = \left.\frac{\delta S}{\delta \Delta\mathcal{H}_A}\right|_{t=0}$.

The curved space action is supersymmetric iff

$$\delta_{\xi,\eta} J_A = -\frac{\delta}{\delta\mathcal{H}_A} \sum_B \left.(J_B, \delta_{\xi,\eta}\mathcal{H}_B)\right|_{t=0} \tag{32}$$

Proof: Step 1

Show that dS/du is exact with respect to $\delta_{\xi,\eta}$

$$\frac{dS}{du} = \delta_{\xi,\eta} \int_{S^3 \times_\Omega S^1_\beta} \bar{J}_{\chi i} \zeta^i \tag{33}$$

where the spinor ζ^i must obeys

$$\begin{aligned}
\bar{\xi}_i \gamma_\mu \zeta^i &= \frac{d}{du} A_\mu^{U(1)_R} \\
2\bar{\xi}_k \gamma^5 \gamma_\mu \zeta^i - \delta_k{}^i \bar{\xi}_j \gamma^5 \gamma_\mu \zeta^j &= \frac{d}{du} A_\mu^{SU(2)_R}{}^i{}_k \\
-4\mathrm{i}\bar{\xi}_i \gamma^5 \gamma_{\mu\nu} \zeta^i &= \frac{d}{du} T_{\mu\nu} \\
2\mathrm{i}\bar{\eta}_i \gamma^5 \zeta^i &= \frac{d}{du} D
\end{aligned} \tag{34}$$

$$
\begin{aligned}
\bar{\xi}_i \gamma_\mu \zeta^i &= \beta \delta_\mu{}^t \\
2\bar{\xi}_k \gamma^5 \gamma_\mu \zeta^i - \delta_k{}^i \bar{\xi}_j \gamma^5 \gamma_\mu \zeta^j &= \frac{1}{2} \omega_\mu{}^{ab} \sigma_{ab} + i(\epsilon_1 + \epsilon_2) \sigma_3 \delta_\mu{}^t \\
-4\mathrm{i} \bar{\xi}_i \gamma^5 \gamma_{\mu\nu} \zeta^i &= 4(dv)^+ \\
2\mathrm{i} \bar{\eta}_i \gamma^5 \zeta^i &= \frac{1}{3}(1-u)R
\end{aligned}
$$

Against all odds, those equations admit a solution

$$
\begin{pmatrix} (\zeta_+)^{\alpha i} \\ (\zeta_-)^{\dot{\alpha} i} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\mathrm{i}(\sigma_2)^{\alpha i} \\ v^\mu (\sigma_\mu \sigma_2)^{\dot{\alpha} i} \end{pmatrix} \Bigg|_{\epsilon_1 \leftrightarrow \epsilon_2}
$$

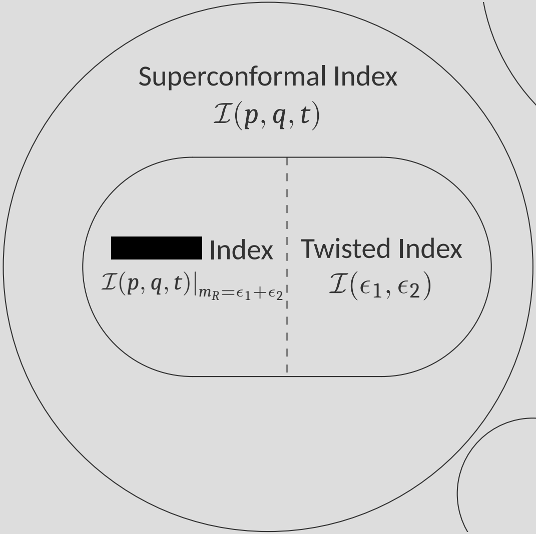
Proof: Step 2

Since $\frac{dS}{du} = \delta_{\xi,\eta}\mathcal{O}$ and $\delta_{\xi,\eta}S = 0$,

$$\begin{aligned}
\frac{d\mathcal{I}}{du} &= \frac{d}{du}\int \mathcal{D}\Phi e^{-S[\Phi,\mathcal{H};u]} \\
&= -\int \mathcal{D}\Phi\, \delta_\xi \mathcal{O}\, e^{-S[\Phi,\mathcal{H};u]} \\
&= -\int \mathcal{D}\Phi\, \delta_\xi \left(\mathcal{O}\, e^{-S[\Phi,\mathcal{H};u]}\right) \\
&= 0
\end{aligned}$$

$\blacksquare$

Result


Superconformal Index
$\mathcal{I}(p,q,t)$
[redacted] Index
$\mathcal{I}(p,q,t)|_{m_R=\epsilon_1+\epsilon_2}$
Twisted Index
$\mathcal{I}(\epsilon_1,\epsilon_2)$

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Summary of Results

1. Constructed a supersymmetric background which interpolates between the [redacted] index and the twisted index
2. Demonstrated the exactness of the interpolation, allowing us to equate the indices
3. In fact, all partition functions at any value of u match
4. Constructed a general formula for the susy variation of the current multiplet in terms of that of the background sugra fields

Honorable mentions

1. Three-sphere squashing and primary Hopf-surfaces [Closset Dumitrescu Festuccia Komargodski 2014]
2. Localization computations?
3. Interpolation between the full SCI and the twisted index?

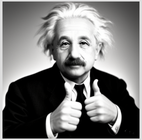
$$\mathcal{I}_{[redacted]} = \mathcal{I}_{\text{twisted}} \tag{35}$$

What does this result imply?

1. Holographically: BH microstate counting?
   - Kerr-Newman BH with angular momentum and electric charge
   - Rotating BH with magnetic R-charge
2. Holographically: exactness $\Rightarrow$ interpolating sugra solution??

An exact interpolating index for 4d $\mathcal{N} = 2$ SCFTs

Thank you for your attention



Relating susy generators after frame rotation

$$F_{ij} = \frac{1}{4}\mathrm{tr}(\gamma_i R \gamma_j R) \qquad F \in SO(4), R \in \mathsf{Spin}(4) \tag{36}$$

$$\xi^{(1)}_{\mathbb{R}^4} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \xi^{(2)}_{\mathbb{R}^4} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \xi^{(3)}_{\mathbb{R}^4} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \xi^{(4)}_{\mathbb{R}^4} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\xi^{(5)}_{\mathbb{R}^4} = -x_a \gamma^a \xi^{(1)} \quad \xi^{(6)}_{\mathbb{R}^4} = -x_a \gamma^a \xi^{(2)} \quad \xi^{(7)}_{\mathbb{R}^4} = -x_a \gamma^a \xi^{(3)} \quad \xi^{(8)}_{\mathbb{R}^4} = -x_a \gamma^a \xi^{(4)}$$

Relating susy generators after frame rotation

$$
\begin{aligned}
e^1 &= dx_1 = e^{-t\beta}\cos(\varphi)\sin(\theta)d\varphi + e^{-t\beta}\sin(\varphi)\cos(\theta)d\theta - \beta e^{-t\beta}\sin(\varphi)\sin(\theta)dt \\
e^2 &= dx_2 = -e^{-t\beta}\sin(\varphi)\sin(\theta)d\varphi + e^{-t\beta}\cos(\varphi)\cos(\theta)d\theta - \beta e^{-t\beta}\cos(\varphi)\sin(\theta)dt \\
e^3 &= dx_3 = -e^{-t\beta}\cos(\theta)\sin(\tau)d\tau - e^{-t\beta}\cos(\tau)\sin(\theta)d\theta - \beta e^{-t\beta}\cos(\theta)\cos(\tau)dt \\
e^4 &= dx_4 = -e^{-t\beta}\sin(\tau)\sin(\theta)d\tau + e^{-t\beta}\cos(\tau)\cos(\theta)d\theta - \beta e^{-t\beta}\sin(\tau)\sin(\theta)dt
\end{aligned}
$$

$$
R = \begin{pmatrix} e^{-\mathrm{i}\tau}\cos\theta & e^{-\mathrm{i}\varphi}\sin\theta & 0 & 0 \\ -e^{\mathrm{i}\varphi}\sin\theta & e^{\mathrm{i}\tau}\cos\theta & 0 & 0 \\ 0 & 0 & \mathrm{i} & 0 \\ 0 & 0 & 0 & -\mathrm{i} \end{pmatrix} \in Spin(4)
$$

Abstract

In this talk I will present some ongoing work on indices of 4d $\mathcal{N}=2$ SCFTs. By coupling a flat-space supersymmetric theory to a rigid supergravity background, we may define its curved space analogue, while preserving some supersymmetry. The partition function of such a theory placed on $S^3 \times S^1$ determines an index with fugacities related to the various background supergravity fields. In the first part of the talk, I will briefly review how this construction works for 4d $\mathcal{N}=2$ SCFTs. With a suitable choice of supergravity background fields, I will show how to recover the superconformal index and the twisted index. In the second part of the talk, I will construct a so-called "interpolating" supergravity background which takes us from the superconformal to the twisted configuration, while still preserving one supercharge. I will further show that this interpolation is in fact exact and the resulting partition function is independent of the interpolating parameter. This allows us to identify the superconformal index with the twisted one. I will conclude by commenting on that identification of the indices.