Machine Learning Toric Duality in Brane Tilings

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Why brane tilings?

- D3-branes probing a singular toric Calabi-Yau
  - ↪ Infinite family of AdS/CFT examples
  - ↪ Admit a graphical representation: bipartite graph on T^2
- Seiberg dualities
  - ↪ act in a controllable manner: urban renewal on the graph
  - ↪ partition the space of dimer models into conjugacy classes

A full classification of brane tilings is still lacking... **ML can help!**

Why ML?

1 Motivation

- Proven useful in classifying tasks
- May help find hidden symmetries in the space of brane tilings
  - ↪ can be used to focus on unexplored theories

Our first steps: **use ML to classify orbifolds of the conifold theories**.

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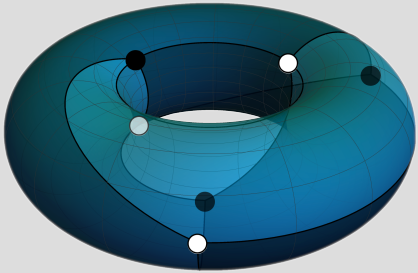
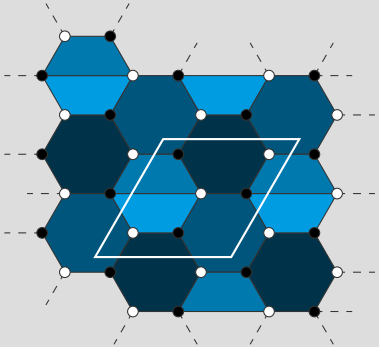
Brane tiling of the dP_1 theory

2 Chosing the data



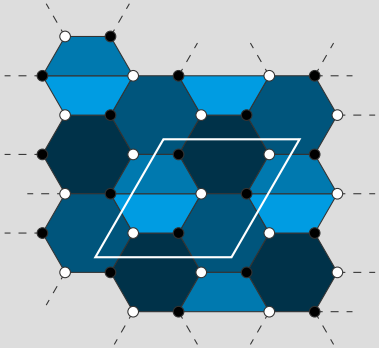
$T^2 \to \mathbb{R}^2$



What is our data?

2 Chosing the data

What do we input into the ML framework?
(or how do we represent the tiling digitally?)

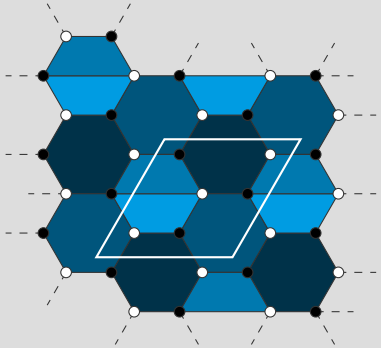


What is our data?

2 Chosing the data

What do we input into the ML framework?
(or how do we represent the tiling digitally?)

- A picture of the universal covering?
- The positions of nodes in the fundamental domain?
- A matrix encoding the node adjacencies?



What is our data?

2 Chosing the data

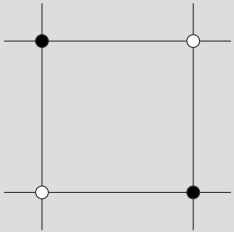
We chose **the Kasteleyn matrix**: a weighted, signed, adjacency matrix of the brane tiling.

Why?

- Its determinant gives the Newton polytope of the underlying tCY 3-fold
  - $\hookrightarrow$ can be used to determine the toric phase
  - $\hookrightarrow$ well behaved under redinition of the fundamental domain
- Up to factors of torus cycles variables, is tensorial and digital $\rightarrow$ well suited for ML
- Action of Seiberg duality is straightforward

Kasteleyn matrix and urban renewal

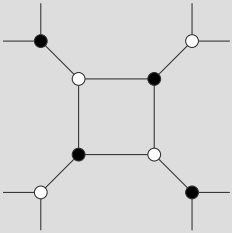
2 Chosing the data



$$\left(\begin{array}{c|cc} A_{N\times N} & B_{N\times 2} \\ \hline C_{2\times N} & \begin{matrix} a & b \\ c & d \end{matrix} \end{array}\right)$$

Kasteleyn matrix and urban renewal

2 Chosing the data

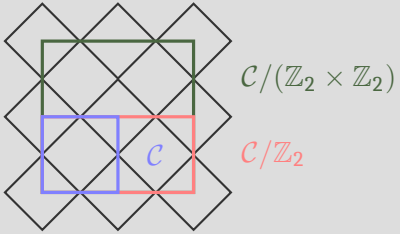


$$\left(\begin{array}{c|c|cc} A_{N\times N} & B_{N\times 2} & 0_{N\times 2} & \\ \hline C_{2\times N} & 0_{2\times 2} & 1 & 0 \\ & & 0 & 1 \\ \hline 0_{2\times N} & \begin{matrix} 0 & -1 \\ -1 & 0 \end{matrix} & \begin{matrix} b & d \\ a & c \end{matrix} & \end{array}\right)$$

The Conifold and its orbifolds

2 Chosing the data

For abelian orbifolds of the conifold the tiling is simply a brane diamond. The action of $\mathbb{Z}_m \times \mathbb{Z}_n$ indicates how many dimaonds the fundamental domain encloses.



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How to deal with formal variables?

3 Constructing a dataset

The Kasteleyn matrix contains two formal variables: w, z
$\hookrightarrow$ they encode how the edge wraps the torus cycles

How to deal with formal variables?

3 Constructing a dataset

The Kasteleyn matrix contains two formal variables: w, z
$\hookrightarrow$ they encode how the edge wraps the torus cycles

$$
\begin{aligned}
1 &\longrightarrow (1,0,0,0,0,0,0,0,0),\\
w &\longrightarrow (0,1,0,0,0,0,0,0,0),\\
1/w &\longrightarrow (0,0,1,0,0,0,0,0,0),\\
z &\longrightarrow (0,0,0,1,0,0,0,0,0),\\
1/z &\longrightarrow (0,0,0,0,1,0,0,0,0),\\
zw &\longrightarrow (0,0,0,0,0,1,0,0,0),\\
z/w &\longrightarrow (0,0,0,0,0,0,1,0,0),\\
w/z &\longrightarrow (0,0,0,0,0,0,0,1,0),\\
1/(wz) &\longrightarrow (0,0,0,0,0,0,0,0,1).
\end{aligned}
\tag{1}
$$

Things to keep in mind

3 Constructing a dataset

We have made dangerous choices

- The Kasteleyn matrix has a large "gauge freedom"
  - ↪ any relabelling of the nodes will permute the rows and columns
  - ↪ any $SL(2,\mathbb{Z})$ redefinition of the cycles will change the w and z content

which turn out to be helpful...

- Urban renewal takes a simple algorithmic form on our tensorial representation of the Kasteleyn matrix

but can become troublesome.

- Urban renewal increases the matrix size
- and may increase the powers of w, and w beyond the assumed set $\{-1, 0, 1\}$

Things to keep in mind

So how do we deal with these problems?

- The Kasteleyn matrix has a large “gauge freedom”
- Urban renewal increases the matrix size
- and may increase the powers of w, and w beyond the assumed set $\{-1, 0, 1\}$

Things to keep in mind

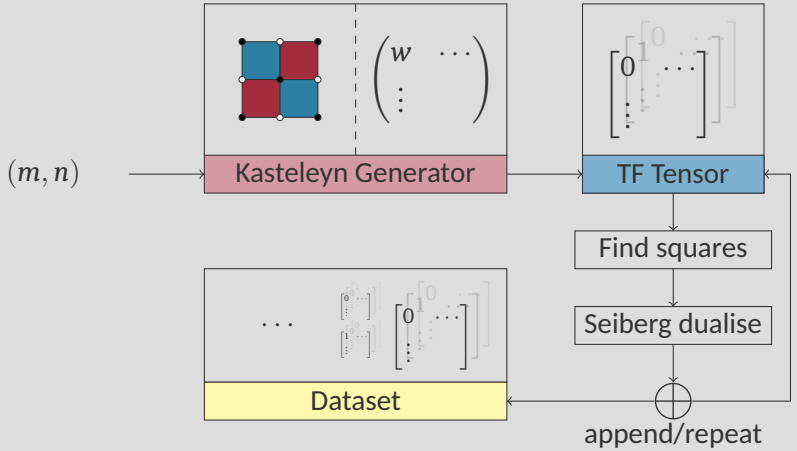
3 Constructing a dataset

So how do we deal with these problems?

- The Kasteleyn matrix has a large "gauge freedom"
  - ↪ Shuffle fixed constructs (rows, column, powers of z, w, etc)
- Urban renewal increases the matrix size
  - ↪ Pad Kasteleyn matrix size
- and may increase the powers of w, and w beyond the assumed set $\{-1, 0, 1\}$
  - ↪ Restrict urban renewal to faces that lie strictly within the fundamental domain.

How can one build the dataset?

3 Constructing a dataset



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How to choose the architecture?

4 Chosing an architecture

We must consider

- The type of data use: here tensors of 0 and 1, of the same shape
- The type of output data: here the abelian orbifold actions (m, n)
- The symmetries of the problem: here row/column flips etc
- The overall complexity of the problem: here simple algebraic function from K to (m, n)

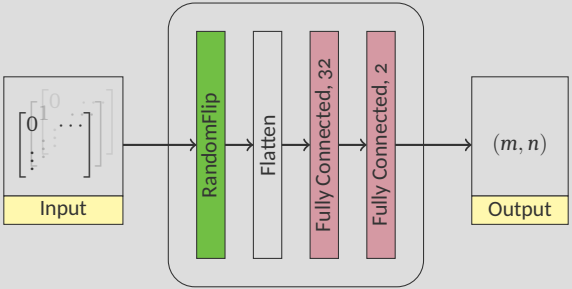
We get two candidates

- The (standard) dense neural network
- The convolutional neural network

The Dense Neural Network

4 Chosing an architecture

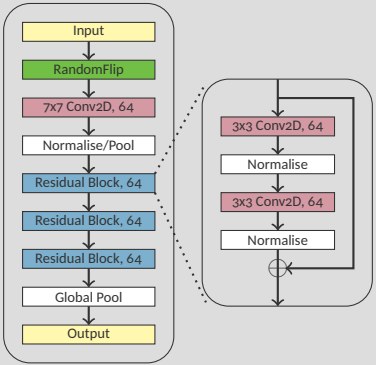
The simplest network is fast to train (good for a proof of concept)



The Residual Neural Network

4 Chosing an architecture

The convolutional neural network can be augmented. This allows to train on more complicated ML data (eg. toric phase classification).



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5 Results

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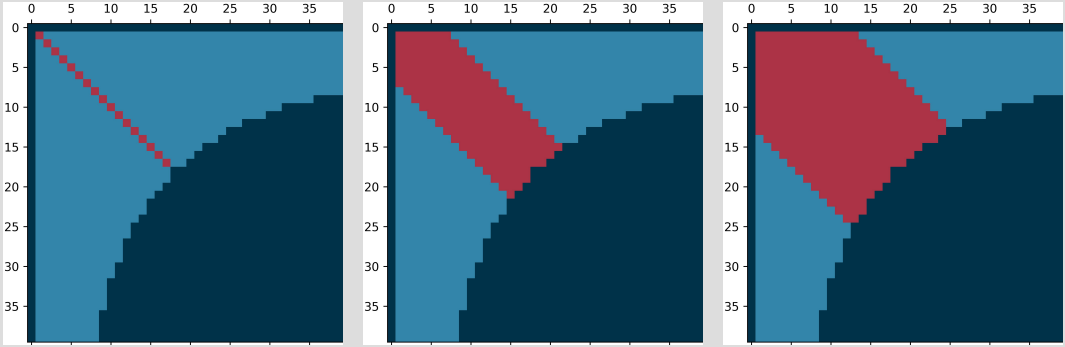
With 50 training epochs and 324,511 matrices of size 128×128

metric \ hparams	ReLU (64, 64)	Sigmoid (32, 32)	Leaky ReLU (64, 64)	Softmax (128, 128)	Linear (128, 64)
R^2	0.95	0.94	0.92	0.85	0.21
MAE	0.95	1.17	1.43	1.72	5.67

Testing robustness

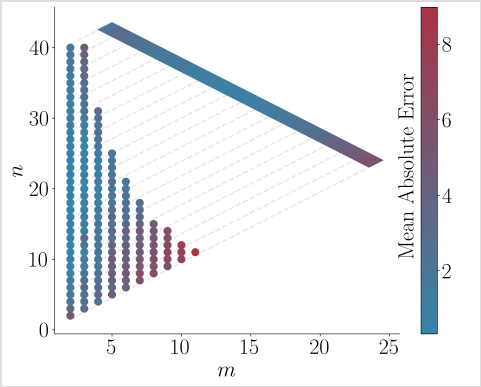
5 Results

We can remove parts of the dataset and perform training again.

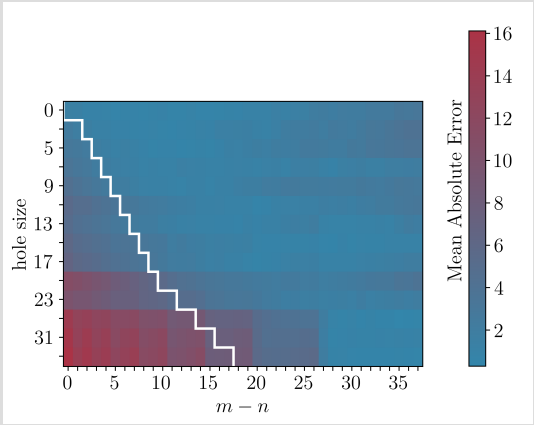


Testing robustness

5 Results



(a)



(b)

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What was achieved?

6 What next?

We successfully trained neural networks to classify abelian orbifolds of the conifold theories, using their Kasteleyn matrices as input (after 450 epochs, $R^2 = 0.988$ and $MAE = 0.72$).

The training is robust under removing parts of the dataset.

The proof of concept was achieved!

What next?

6 What next?

This must be extended to a larger family of brane tiling.

- One could use graph neural networks (help reduce gauge freedom in dataset)
- Attempt to predict invariants of the theories (SCI, Hilbert series, etc)