Conformal Defects in Neural Network Field Theories

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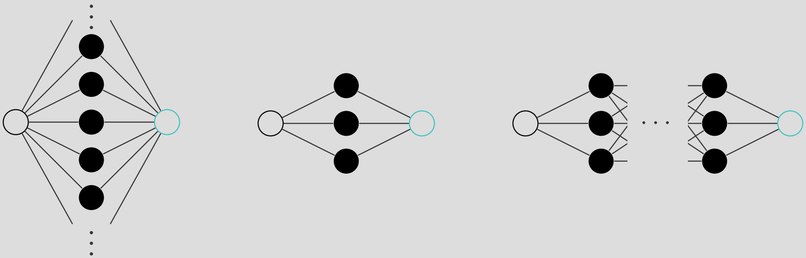
What is a neural-network QFT and why do we care?

Neural Network: a map $f : \mathbb{R}^m \to \mathbb{R}^n$

Operations: composition, addition, scalar multiplication

Universal Approximation Theorems (UATs): the NNs are dense in a space of maps (in some limit).



$\hookrightarrow$ Depends on choice of basis, and choice of infinite-node limit.

What is a neural-network QFT and why do we care?

QFT: Generating functional: $Z[J] = \int \mathcal{D}\phi \, e^{-S[\phi]+\int d^D x \phi(x) J(x)}$.

Key Idea: Approximate the functional integral using the UATs and NNs.[Halverson, Maiti and Stoner; 2008]

NNs come with a set of parameters: eg. Sigmoids $= \{\sigma(a \cdot x + b)\}$.
Integrate over those with a measure:

$$\int \mathcal{D}\phi \, e^{-S[\phi]} \quad \longleftrightarrow \quad \lim_{N\to\infty} \int \prod_i^N d\theta_i P(\theta_i) \tag{1}$$

In the limit where the UAT applies, both sides are equal.

Result: Novel way of constructing QFT correlators.

What about neural-network CFTs?

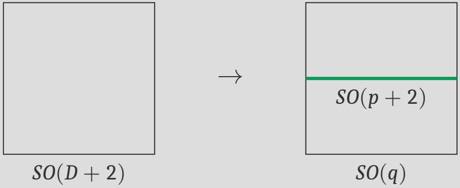
CFT: More symmetry $\to$ more constrained dynamics.
Full CFT data encoded in set of numbers (Δ_i, λ_{ij}).

NN-CFT: Approximation is exact at any number of neurons! No need to take a UAT limit!
[Halverson, Naskar, Tian; 2024]

Conformal primary	
$\phi(x) \mapsto \lambda^{-\Delta}\phi(x)$	$\Phi(X) \mapsto \lambda^{-\Delta}\Phi(X)$
Partition function	
$Z[J] = \int \mathcal{D}\phi e^{-S[\phi]} e^{\int d^dx J(x)\phi(x)}$	$Z[J] = \int d\theta \mathcal{P}(\theta) e^{\int d^dx J(x)\Phi(x)}$
Correlators	
$\langle \phi(x)\phi(y) \rangle$	$\mathbb{E}[\Phi(x)\Phi(y)]$

Extending this to theories with defects

Break the symmetry group: $SO(D+2) \to SO(p+2) \times SO(q)$



dCFT: Full data encoded in set of numbers $(\hat{\Delta}_i, \Delta_i, \lambda_{ij}, \lambda_{ijk})$

Result: Neural-network approximation is still exact at finite number of neurons! [Capuozzo, Robinson, Suzzoni; 2025]

We can build new defect CFTs simply by specifying an invariant distribution and conformal primaries fields!

Conclusion

What was known

1. Neural-Network CFTs can be engineered from a finite number of building blocks (neurons)
2. Correlators are well defined (standard statistical averages)
3. Can combine neurons to form new CFTs! $\to$ playground for generating CFT data

What we did

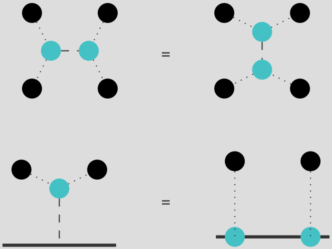
1. Extended the notion of nn-CFT to the defect case
2. Constructed simple examples of fully solvable nn-dCFTs

Formalism doesn't yet exist for spin $\neq \mathbf{0}$ $\hookrightarrow$ Important for condensed matter, string theory, AdS/CFT, ...

Thank you

dCFT correlator constraints

3 Conclusion



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